

Charged lepton contributions to bimaximal and tri-bimaximal mixing for generating $\sin \theta_{13} \neq 0$ and $\tan^2 \theta_{23} < 1$

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Abstract

Bimaximal(BM) and Tri-bimaximal(TB) mixings of neutrinos are two special cases of lepton mixing matrix, which predict the reactor angle $\theta_{13} = 0$ and the atmospheric angle $\tan^2 \theta_{23} = 1$. Recent precision measurements and global analysis of oscillation parameters, have confirmed a non-vanishing value of θ_{13} as well as deviations of θ_{12} and θ_{23} from their maximal values predicted by BM or TB mixing. In this work we mainly concentrate on θ_{13} and θ_{23} to assign $\theta_{13} \neq 0$ and $\tan^2 \theta_{23} < 1$ with the help of charged lepton corrections defined by $U_{PMNS} = U_l^\dagger U_\nu$. We first consider U_ν to be given separately by BM and TB mixing matrices and then find the possible forms of U_l such that the elements of PMNS matrix, finally yield $\theta_{13} \neq 0$ and $\tan^2 \theta_{23} < 1$ in agreement with latest observational data. To compute the values of mixing angles we assume the charged lepton correction to be of Cabbibo-Kobayashi-Maskawa(CKM) like. All the mixing matrices involved in the calculation satisfy the unitarity condition to leading order of expansion parameter.

Key-words: Charged lepton correction, Bimaximal and Tri-bimaximal mixing.

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1 Introduction

Recent precision measurements[1-4] and latest global 3ν oscillation analysis[5] of neutrino mixing parameters, have confirmed non-vanishing value of θ_{13} as well as deviation of atmospheric mixing angle from maximal value, $\theta_{23} < \pi/4$. One of the important aspects of neutrino physics is to understand such mixing patterns[6]. Charged lepton correction to neutrino mixing matrix is an attractive tool which can impart non-zero value of θ_{13} as well as deviation of θ_{23} from maximal value. We address the issue of charged lepton correction to both bimaximal(BM) and tri-bimaximal(TB) neutrino mixings to produce desired results.

To begin with we start with the lepton mixing matrix, known as Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix[7],

$$U_{PMNS} = U_l^\dagger U_\nu, \quad (1)$$

which is analogous to CKM matrix, $V_{CKM} = U_{uL}^\dagger U_{dL}$ for quark sector[8,9]. In relation (1), U_l and U_ν are the diagonalizing matrices for charged lepton and left-handed Majorana neutrino mass matrices respectively which are defined as : $m_l = U_{lL} m_l^{diag} V_{lR}^\dagger$ and $m_\nu = U_\nu^* m_\nu^{diag} U_\nu^\dagger$. In the basis where charged lepton mass matrix is diagonal, m_ν is expressible as[10]

$$m'_\nu = U_{lL}^\dagger m_\nu U_{lL}. \quad (2)$$

In the standard Particle Data Group (PDG) parametrization[9], with three mixing angles and three CP phases- one Dirac CP phase (δ) and two Majorana CP phases (α, β), PMNS matrix has the form,

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} . P, \quad (3)$$

where $c_{ij} = \cos \theta_{ij}$, $s_{ij} = \sin \theta_{ij}$ with θ_{12} being the solar angle, θ_{23} being the atmospheric angle and θ_{13} being the reactor angle and $P = diag(1, e^{i\alpha}, e^{i\beta})$ contains the Majorana CP phases. In our present work we ignore all the CP phases. Then under $\mu - \tau$ symmetry, with $\theta_{13} = 0$, PMNS matrix takes the form[11] :

$$U_{PMNS} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -\frac{s_{12}}{\sqrt{2}} & \frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{s_{12}}{\sqrt{2}} & -\frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad (4)$$

which predicts maximal value of the atmospheric angle ($\theta_{23} = \frac{\pi}{4}$) leaving solar angle (θ_{12}) arbitrary.

Two popular neutrino mixing matrices are the bi-maximal[12] mixing (BM) and the tri-bimaximal (TB) mixing[13], which can be obtained from eq.(4) by setting $s_{12} = \frac{1}{\sqrt{2}}$ and $s_{12} = \frac{1}{\sqrt{3}}$ respectively and are given as:

$$U_{BM} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad U_{TB} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \sqrt{\frac{1}{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \quad (5)$$

Both these two neutrino mixing matrices predict $\tan^2 \theta_{23} = \frac{|U_{\mu 3}|^2}{|U_{\tau 3}|^2} = 1$ and $\sin^2 \theta_{13} = |U_{e 3}|^2 = 0$.

The paper is organized as follows: In section 2 we discuss charged lepton correction to BM neutrino mixing and present predictions of the mixing angles along with graphical representations. In a similar way section 3 is devoted to TB mixing and finally section 4 is devoted to summary and discussion.

2 Charged lepton correction to BM mixing

General forms of the lepton mixing matrix (U_ν) and the neutrino mixing matrix (U_l) in equation(1) can be expressed as

$$U_l = \begin{pmatrix} c_{12}^l c_{13}^l & s_{12}^l c_{13}^l & s_{13}^l \\ -s_{12}^l c_{23}^l - c_{12}^l s_{23}^l s_{13}^l & c_{12}^l c_{23}^l - s_{12}^l s_{23}^l s_{13}^l & s_{23}^l c_{13}^l \\ s_{12}^l s_{23}^l - c_{12}^l c_{23}^l s_{13}^l & -c_{12}^l s_{23}^l - s_{12}^l c_{23}^l s_{13}^l & c_{23}^l c_{13}^l \end{pmatrix} \quad (6)$$

and

$$U_\nu = \begin{pmatrix} c_{12}^\nu c_{13}^\nu & s_{12}^\nu c_{13}^\nu & s_{13}^\nu \\ -s_{12}^\nu c_{23}^\nu - c_{12}^\nu s_{23}^\nu s_{13}^\nu & c_{12}^\nu c_{23}^\nu - s_{12}^\nu s_{23}^\nu s_{13}^\nu & s_{23}^\nu c_{13}^\nu \\ s_{12}^\nu s_{23}^\nu - c_{12}^\nu c_{23}^\nu s_{13}^\nu & -c_{12}^\nu s_{23}^\nu - s_{12}^\nu c_{23}^\nu s_{13}^\nu & c_{23}^\nu c_{13}^\nu \end{pmatrix}, \quad (7)$$

where we have ignored the CP violating phases. For our case we first consider the neutrino mixing pattern to be of bi-maximal nature. Then $U_\nu = U_{BM}$

is given by eq.(5). We then take the following form of the lepton mixing matrix[14],

$$U_l = \begin{pmatrix} \tilde{c}_{12} & \tilde{s}_{12} & 0 \\ -\tilde{s}_{12} & \tilde{c}_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (8)$$

where $\tilde{s}_{ij} = \sin \theta_{ij}^l$ and $\tilde{c}_{ij} = \cos \theta_{ij}^l$. This structure(8) had been studied earlier[14] but we study it again here in the light of latest observational data[5].

From eqs.(1), (5) and (8), we finally obtain the PMNS matrix $U_{PMNS} = U_l^\dagger U_{BM}$ as

$$U_{PMNS} = \begin{pmatrix} \frac{1}{\sqrt{2}}(\tilde{c}_{12} + \frac{\tilde{s}_{12}}{\sqrt{2}}) & \frac{1}{\sqrt{2}}(\tilde{c}_{12} - \frac{\tilde{s}_{12}}{\sqrt{2}}) & -\frac{\tilde{s}_{12}}{\sqrt{2}} \\ -\frac{1}{2}(\tilde{c}_{12} - \sqrt{2}\tilde{s}_{12}) & \frac{1}{2}(\tilde{c}_{12} + \sqrt{2}\tilde{s}_{12}) & \frac{\tilde{c}_{12}}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}. \quad (9)$$

Let us now assume that the charged lepton corrections are Cabbibo-Kobayashi-Maskawa (CKM) like[9], which allows us to take

$$\tilde{s}_{12} = \sin \theta_{12}^l = \lambda, \quad (10)$$

where the Wolfenstein parameter λ is related to the Cabbibo angle (θ_C) by $\lambda = \sin \theta_C$. Under this consideration, PMNS matrix in eq.(9), can be approximated to the form,

$$U_{PMNS} \approx \begin{pmatrix} \frac{1}{\sqrt{2}}(1 + \frac{\lambda}{\sqrt{2}} - \frac{\lambda^2}{2}) & \frac{1}{\sqrt{2}}(1 - \frac{\lambda}{\sqrt{2}} - \frac{\lambda^2}{2}) & -\frac{\lambda}{\sqrt{2}} \\ -\frac{1}{2}(1 - \sqrt{2}\lambda - \frac{\lambda^2}{2}) & \frac{1}{2}(1 + \sqrt{2}\lambda - \frac{\lambda^2}{2}) & \frac{1}{\sqrt{2}}(1 - \frac{\lambda^2}{2}) \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}. \quad (11)$$

And the expression in eq.(8) becomes

$$U_{lL} = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & -\lambda & 0 \\ \lambda & 1 - \frac{\lambda^2}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (12)$$

It can be emphasised here that both mixing matrices in eqs.(11) and (12) satisfy the unitarity condition as expected. Then eq.(11) leads to

$$\tan^2 \theta_{12} = \left(\frac{1 - U_{e3} - U_{e3}^2}{1 + U_{e3} - U_{e3}^2} \right)^2, \quad (13)$$

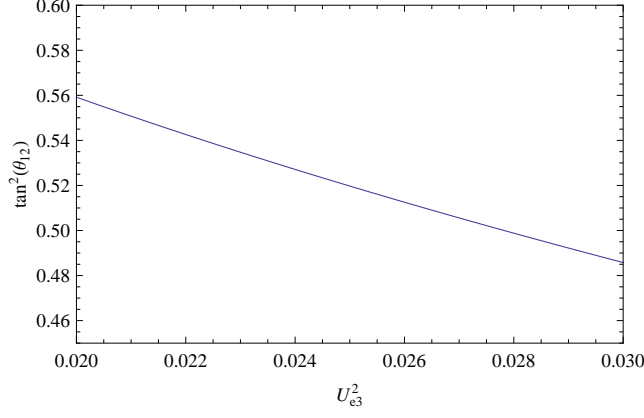


Figure 1: Variation of $\tan^2 \theta_{12}$ with U_{e3}^2 for BM mixing after taking charged lepton correction

$$\tan^2 \theta_{23} = (1 - U_{e3}^2)^2, \quad (14)$$

$$U_{e3}^2 = \sin^2 \theta_{13} = \frac{\lambda^2}{2}. \quad (15)$$

With $\lambda = 0.232$ corresponding to $|U_{e3}|^2 = 0.027$, we get $\tan^2 \theta_{12} \approx 0.50$ and $\tan^2 \theta_{23} = 0.946$. The variation of $\tan^2 \theta_{12}$ with $|U_{e3}|^2$ is shown in Fig.1 and that of $\tan^2 \theta_{23}$ with $|U_{e3}|^2$ is shown in Fig.2 .

3 Charged lepton correction to TB Mixing

Tri-bimaximal neutrino mixing is a special case of mixing matrix with $\mu - \tau$ symmetry. It can give a very close description of the experimental data except the case: $\theta_{13} = 0$. The TB neutrino mixing matrix ($U_\nu = U_{TB}$) is given in equation (5). In order to account for the charged lepton correction to the TB neutrino mixing, we start with the lepton mixing matrix which satisfies unitarity condition,

$$\tilde{U}_l = \begin{pmatrix} 1 - \frac{\lambda^2}{4} & -\frac{\lambda}{2} & -\frac{\lambda}{2} \\ \frac{\lambda}{2} & 1 - \frac{\lambda^2}{8} & -\frac{\lambda^2}{8} \\ \frac{\lambda}{2} & -\frac{\lambda^2}{8} & 1 - \frac{\lambda^2}{8} \end{pmatrix}. \quad (16)$$

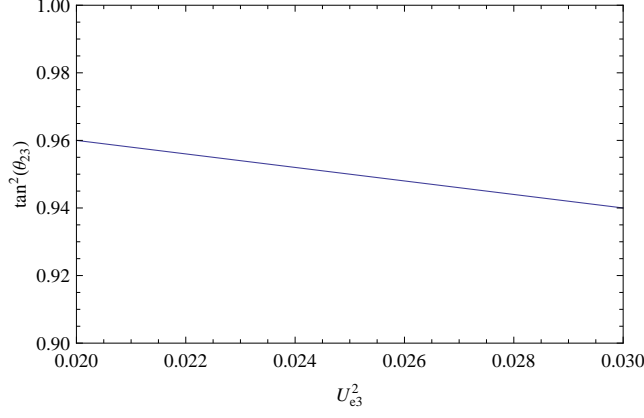


Figure 2: Variation of $\tan^2 \theta_{23}$ with U_{e3}^2 for BM mixing after taking charged lepton correction

Using the form of U_ν for TB, given by eq.(5), we have $U_{PMNS} = \tilde{U}_l^\dagger U_{TB}$ which reproduces the following PMNS matrix first proposed by King[15],

$$U_{PMNS} = \begin{pmatrix} \sqrt{\frac{2}{3}}(1 - \frac{\lambda^2}{4}) & \frac{1}{\sqrt{3}}(1 - \frac{\lambda^2}{4}) & \frac{\lambda}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}}(1 + \lambda) & \frac{1}{\sqrt{3}}(1 - \frac{\lambda}{2}) & \frac{1}{\sqrt{2}}(1 - \frac{\lambda^2}{4}) \\ \frac{1}{\sqrt{6}}(1 - \lambda) & -\frac{1}{\sqrt{3}}(1 + \frac{\lambda}{2}) & \frac{1}{\sqrt{2}}(1 - \frac{\lambda^2}{4}) \end{pmatrix}. \quad (17)$$

This PMNS matrix has unique property of unitarity to leading order, and also predicts $\tan^2 \theta_{23} = 1$. In order to have $\tan^2 \theta_{23} < 1$ in the light of present experimental data[5], we now modify the charged lepton mixing matrix(16) by the relation

$$U_l^\dagger = \tilde{R}_{23}^\dagger \tilde{U}_l^\dagger, \quad (18)$$

where \tilde{R}_{23} has a structure similar to that of rotation matrix and is given by

$$\tilde{R}_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \tilde{c}_{23} & \tilde{s}_{23} \\ 0 & -\tilde{s}_{23} & \tilde{c}_{23} \end{pmatrix}, \quad (19)$$

with $\tilde{s}_{23} = \sin \theta_{23}^l$ and $\tilde{c}_{23} = \cos \theta_{23}^l$.

Then equations (1),(16) and (18) give the following elements of the new PMNS matrix, $U_{PMNS} = U_l^\dagger U_{TB}$,

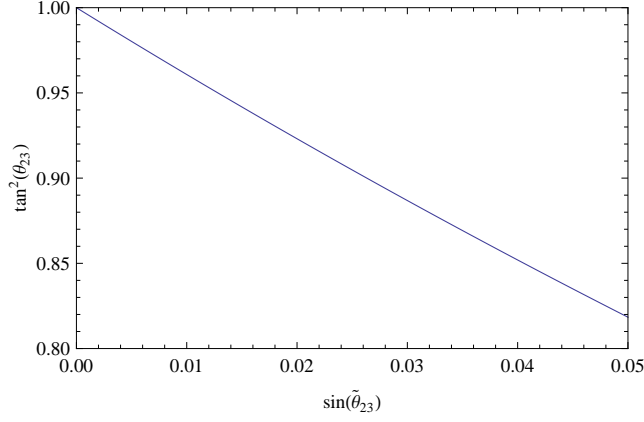


Figure 3: Variation of $\tan^2 \theta_{23}$ with $\sin \tilde{\theta}_{23}$ for TB mixing after taking charged lepton correction

$$\begin{aligned}
(U_{PMNS})_{11} &= \sqrt{\frac{2}{3}} \left(1 - \frac{\lambda^2}{4}\right), \\
(U_{PMNS})_{12} &= \frac{1}{\sqrt{3}} \left(1 - \frac{\lambda^2}{4}\right), \\
(U_{PMNS})_{13} &= \frac{\lambda}{\sqrt{2}}, \\
(U_{PMNS})_{21} &= -\frac{1}{\sqrt{6}} [(\tilde{c}_{23} + \tilde{s}_{23}) + (\tilde{c}_{23} - \tilde{s}_{23})\lambda], \\
(U_{PMNS})_{22} &= \frac{1}{\sqrt{3}} [(\tilde{c}_{23} + \tilde{s}_{23}) - (\tilde{c}_{23} - \tilde{s}_{23})\frac{\lambda}{2}], \\
(U_{PMNS})_{23} &= \frac{1}{\sqrt{2}} (\tilde{c}_{23} - \tilde{s}_{23}) \left(1 - \frac{\lambda^2}{4}\right), \\
(U_{PMNS})_{31} &= \frac{1}{\sqrt{6}} [(\tilde{c}_{23} - \tilde{s}_{23}) - (\tilde{c}_{23} + \tilde{s}_{23})\lambda], \\
(U_{PMNS})_{32} &= -\frac{1}{\sqrt{3}} [(\tilde{c}_{23} - \tilde{s}_{23}) + (\tilde{c}_{23} + \tilde{s}_{23})\frac{\lambda}{2}], \\
(U_{PMNS})_{33} &= \frac{1}{\sqrt{2}} (\tilde{c}_{23} + \tilde{s}_{23}) \left(1 - \frac{\lambda^2}{4}\right).
\end{aligned}$$

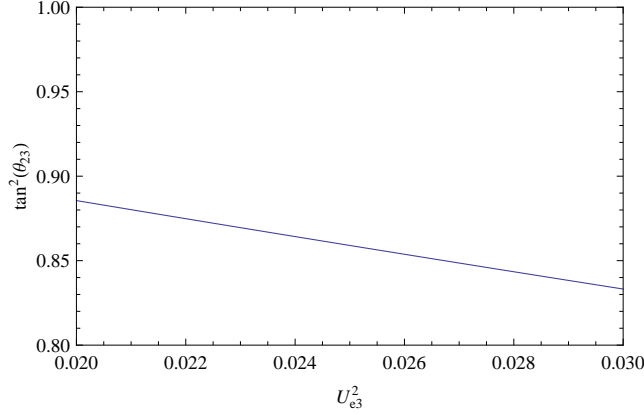


Figure 4: Variation of $\tan^2 \theta_{23}$ with U_{e3}^2 for TB mixing after taking charged lepton correction

From these elements we calculate

$$\tan^2 \theta_{23} = \left(\frac{1 - \tan \tilde{\theta}_{23}}{1 + \tan \tilde{\theta}_{23}} \right)^2, \quad (20)$$

which is lesser than maximal value for non-zero $\tan \tilde{\theta}_{23}$. Assuming that the charged lepton corrections are Cabbibo-Kobayashi-Maskawa (CKM) like, we can have[9,16]

$$\tilde{s}_{23} = \sin \theta_{23}^l = A\lambda^2 \approx 0.041, \quad (21)$$

leading to $\tan^2 \theta_{23} = 0.85$, where we have adopted $\lambda = 0.2324$ and $A = 0.759$. The variations of $\tan^2 \theta_{23}$ with $\sin \tilde{\theta}_{23}$ and $|U_{e3}|^2$ are shown in Fig.3 and Fig.4 respectively.

4 Summary and Discussion

We have studied two possible forms of the lepton mixing matrix U_l which can produce desired deviations from the bimaximal (BM) and tri-bimaximal(TB) mixings of neutrino sector under charged lepton corrections. The lepton mixing matrices have basically been derived from rotation matrices and hence the conditions of unitarity of all diagonalising matrices including the final

form of PMNS matrices discussed here, are satisfied at leading order. In such situation PMNS matrix proposed by King[15] is a pointer to the right direction. Assuming the charged lepton correction is CKM-like, we calculate $\tan^2 \theta_{23} = 0.946 < 1$ for BM case and $\tan^2 \theta_{23} = 0.85 < 1$ for TB case with $\lambda = 0.232$ and $A = 0.759$ and $\sin^2 \theta_{13} = 0.027$ for both BM and TB cases.

The deviation of solar mixing angle $\tan^2 \theta_{12}$ below the value of 0.50, can be introduced in realistic $\mu - \tau$ symmetric neutrino mass matrices with specific choices of value of flavour twister term[14,17,18] present in the texture of the mass matrices, without affecting the good predictions on reactor and atmospheric mixing angles.

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